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MATHEMATICAL MODELING AND MICROMECHANICS OF FIBER-REINFORCED BIMODULUS COMPOSITE MATERIALS

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Norman, Oklahoma

June 1979

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

→ In Part I of this report are described and evaluated various mathematical constitutive equations proposed to model the mechanical behavior of fiber-reinforced composite materials having different compliances in tension and compression. Five different constitutive equations are described in detail and evaluated in the light of three criteria.

In Part II of the report two entirely different micromechanistic →

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approaches are developed to explain these observed differences in tension/compression behavior: the mean-fiber-angle approach originated by Tarnopol'skii et al. and the elastically supported tie-bar/column approach originated by Herrmann et al.

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MATHEMATICAL MODELING AND MICROMECHANICS OF FIBER-REINFORCED BIMODULUS COMPOSITE MATERIALS

by

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June 1979

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PART I.

RECENT ADVANCES IN MATHEMATICAL MODELING OF THE MECHANICS OF BIMODULUS, FIBER-REINFORCED COMPOSITE MATERIALS*

1. INTRODUCTION

The classical elasticity theory hypothesis [1] that the material obeys the generalized Hooke's linear stress-strain relation is justified for application to most metallic alloys loaded below the elastic limit. For more highly flexible materials including polymers, various nonlinear constitutive relations such as neo-Hookean, Mooney-Rivlin, and Ogden have been proposed; see [2]. All of these relations make use of extension ratio (λ) rather than engineering strain (ε); note that $\lambda = 1 + \varepsilon$. Most of the relations have a different elastic modulus at a finite value of compressive strain ($\varepsilon_o < 0$) than at the same absolute value of tensile strain ($|\varepsilon_o| > 0$). However, in agreement with the careful measurements of various investigators (cf. [3]), these finite-strain relations have no discontinuity in slope in going from compressive strain ($\lambda < 1$) to tensile strain ($\lambda > 1$).

In contrast, certain materials demonstrate a distinct change in modulus in going from compression to tension†; see Fig. 1. These materials apparently are primarily composite materials, as listed in Table 1. In the literature, this class of materials has variously been called bilinear, bimodulus, different-modulus, and multimodulus. Here the term bimodulus is believed to be most descriptive of a material having different linear stress-strain relations in compression than in tension.

The first multidimensional model for bimodulus materials was proposed by Ambartsumyan [8] for isotropic material, such as a composite material with spherical particles. It was later extended to the orthotropic case [9].

The second and third models are the restricted compliance model due to Isabekyan and Khachatryan [10] and the first-invariant model of Shapiro [11]. A fourth model is the weighted compliance theory originated by Jones [12].

The fifth model is the fiber-governed bimodulus symmetric compliance model originated by Bert [6].

In the next section, criteria for evaluating bimodulus material models are presented and in subsequent sections the criteria are applied in critically evaluating the various models.

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* This part of the report is a slight expansion of a paper of the same title presented at the 15th Annual Meeting of the Society of Engineering Science, Gainesville, Florida, Dec. 4-6, 1978.

† Saint-Venant [4] in 1864 made perhaps the earliest mention of material with different behavior in tension and compression.

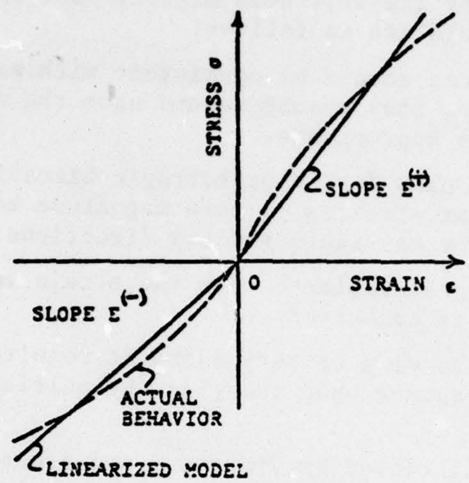

Fig. 1 Bimodulus Idealization

Table 1. Some Bimodulus Materials

Material	Reinforcement Geometry	Ref.	Tensile Young's Modulus Divided by Compressive Young's Modulus
ATJ-S graphite	Granular	5	1.2
ZTA graphite	Granular	5	0.8
Glass-epoxy	Fibrous	5	1.25
Boron-epoxy	Fibrous	5	0.8
Graphite-epoxy	Fibrous	5	1.4
Carbon-carbon	Fibrous	5	2.0 to 5.0
Kevlar-rubber	Fibrous	6	0.77 (transverse) to 305 (longitudinal)*
Polyester-rubber	Fibrous	-	0.75 (transverse) to 16.7 (longitudinal)*

* Based on experimental results reported by Patel et al. [7].

2. CRITERIA FOR EVALUATING MATERIAL MODELS

A linear material model can be characterized by either its compliance matrix or alternatively by its stiffness matrix. The criteria used to evaluate the various material models are as follows:

1. The compliances should be consistent with measured values for the conditions specified, i.e. they should depend upon the nature of the multiaxial stress or strain state as appropriate.

2. The shear modulus for an orthotropic bimodulus material should have different values for shear stresses of same magnitude but opposite sign in any coordinates other than the material-symmetry directions.

3. It would be preferable to have the strain energy be positive definite so that energy is conserved.

Criterion 1 is the main criterion and it requires that the model be able to duplicate measured response upon changing the multiaxial stress or strain state as appropriate.

Criterion 2 is clarified by Figures 2 and 3 due to Jones [5]. In Fig. 2, it is apparent that the fibers are loaded in exactly the same way by positive and negative shear stresses, while in Fig. 3, it is clear that the fibers are loaded in tension by a positive shear stress and in compression by a negative shear stress.

As alluded to by Voigt [13] and shown by Eubanks and Sternberg [14] and Lempriere [15], criterion 3 implies that: (a) the compliance matrix be symmetric and (b) certain limits exist on the compliances so that the compliance matrix is positive definite. Symmetry of the compliance matrix is necessary in order for a material to be mechanically stable, as shown by Brun [16]. Furthermore, compliance symmetry is highly desirable in that most structural analysis algorithms are based on this assumption, i.e., they have no provision for unsymmetric compliance or stiffness matrices.

In the sections to follow only the orthotropic versions of the various material models are presented and these are limited to the plane-stress case for brevity.

3. MODEL I: THE AMBARTSUMYAN MODEL

In this theory [9], the srains are expressed in terms of the stresses as follows

$$\begin{aligned} \epsilon_1 &= b_{11}\,\sigma_1 + b_{12}\,\sigma_2 \\ \epsilon_2 &= b_{21}\,\sigma_1 + b_{22}\,\sigma_2 \\ \epsilon_{12} &= b_{61}\,\sigma_1 + b_{62}\,\sigma_2 \end{aligned} \tag{1}$$

Here σ_1, σ_2 are principal stresses; ϵ_1, ϵ_2, ϵ_{12} are the normal and shear strains associated with principal-stress directions 1, 2 ; b_{ij} are the comliance coefficients which take on values as follows:

$$b_{ij} = \begin{cases} b_{ij}^{(+)} & \text{if } \sigma_j \geq 0 \\ b_{ij}^{(-)} & \text{if } \sigma_j < 0 \end{cases} \tag{2}$$

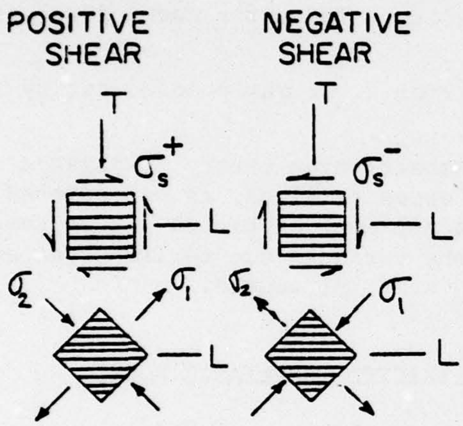

Fig. 2 Positive and negative shear stress applied along material-symmetry directions.

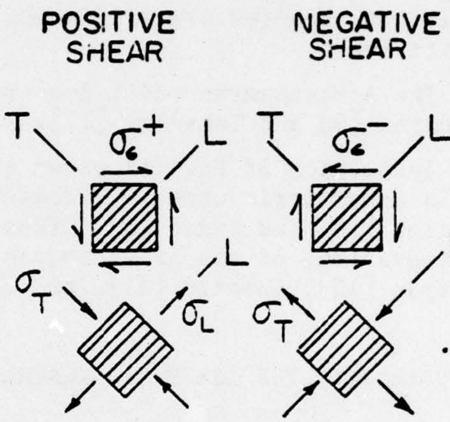

Fig. 3 Positive and negative shear stress applied at 45 degrees to the material-symmetry directions.

In a bimodulus material, $b_{ij}^{(-)} \neq b_{ij}^{(+)}$ by definition, while in a classical material $b_{ij}^{(-)} = b_{ij}^{(+)}$. It is noted that in an orthotropic material the principal-strain directions do not coincide with the principal-stress directions; thus, the presence of ε_{12} is necessary.

It should be noted that the lack of the explicit presence of a shear stress does not mean that the principal-stress directions coincide with the material-symmetry direction. It must be remembered that the principal stresses and their directions depend directly upon the original stress state $(\sigma_x, \sigma_y, \sigma_{xy})$ with respect to arbitrary axes x, y.

The major criticism of the original Ambartsumyan model insofar as it applies to filamentary composite materials, is that it does not relate to material-symmetry directions, which are of major importance due to the different mechanisms of tensile vs. compressive stiffening or softening. Thus, the model does not meet criterion 1.

The Ambartsumyan model does meet criterion 2, as was pointed out by Ambartsumyan [9] and Tabaddor [17].

Inspection of Eq. (1) shows that the Ambartsumyan-theory compliance matrix is unsymmetric when σ_1 and σ_2 have opposite signs, as was pointed out by Tabaddor [18] and Jones [5]. Thus, it does not meet criterion 3. To remedy this disadvantage of the Ambartsumyan model, the variants due to Isabekyan and Khachatryan [10], Shapiro [11], and Jones [12] were introduced.

4. MODEL II: THE ISABEKYAN-KHACHATRYAN RESTRICTED-COMPLIANCE MODEL

This model was introduced by Isabekyan and Khachatryan [10] and is a modification of the Ambartsumyan model in which the following limits are imposed to enforce symmetry of the compliance matrix

$$\begin{aligned} b_{12}^{(+)} &= b_{12}^{(-)} = b_{21}^{(+)} = b_{21}^{(-)} \\ b_{61}^{(+)} &= b_{61}^{(-)} = b_{62}^{(+)} = b_{62}^{(-)} \end{aligned} \tag{3}$$

Model II still has the same disadvantage as the Ambartsumyan theory in that it does not meet criterion 1. In fact, the limitations imposed upon the coefficients in Eqs. (3) make it less likely to meet criterion 1. Also, it does not meet criterion 2, although it does meet criterion 3.

5. MODEL III: THE SHAPIRO FIRST-INVARIANT MODEL

This model was introduced by Shapiro [11] and can be considered to be another variation of the Ambartsumyan model, Eqs. (1), in which now the b_{ij} are determined on the basis of the sign of a combination of stresses in the stress state, namely, the first invariant of stress defined as

$$\sigma_I = (1/3)(\sigma_1 + \sigma_2 + \sigma_3) = (1/3)(\sigma_x + \sigma_y + \sigma_z) \tag{4}$$

For the plane-stress case considered here,

$$\sigma_I = (1/3)(\sigma_1 + \sigma_2) = (1/3)(\sigma_x + \sigma_y) \tag{5}$$

Thus, here

$$b_{ij} = \begin{cases} b_{ij}^{(+)} & \text{if } \sigma_I \geq 0 \\ b_{ij}^{(-)} & \text{if } \sigma_I < 0 \end{cases} \tag{6}$$

Again the major disadvantage of Model III is the same as the Ambartsumyan model in failing to meet criterion 1. Also, like Model II, it fails to meet criterion 2 but does meet criterion 3.

6. THE JONES WEIGHTED-COMPLIANCE MODEL

This model can also be considered to be a variant of the Ambartsumyan model. It was introduced by Jones [12], who elaborated upon it later [5]. For the plane-stress case [5],

$$b_{ij} = \left\{ \begin{array}{ll} b_{ij}^{(+)} & \text{if } \sigma_j \geq 0 \\ b_{ij}^{(-)} & \text{if } \sigma_j < 0 \end{array} \right\} ij = 11,\ 22,\ 61,\ 62$$

$$b_{21} = b_{12} = \begin{cases} b_{12}^{(+)} & \text{if } \sigma_1 > 0 \ \&\ \sigma_2 > 0 \\ k_1 b_{12}^{(+)} + k_2 b_{12}^{(-)} & \text{if } \sigma_1 > 0 \ \&\ \sigma_2 < 0 \\ k_1 b_{12}^{(-)} + k_2 b_{12}^{(+)} & \text{if } \sigma_1 < 0 \quad \sigma_2 > 0 \\ b_{12}^{(-)} & \text{if } \sigma_1 < 0 \ \&\ \sigma_2 < 0 \end{cases} \tag{7}$$

where

$$k_1 \equiv \frac{|\sigma_1|}{|\sigma_1| + |\sigma_2|}, \quad k_2 \equiv \frac{|\sigma_2|}{|\sigma_1| + |\sigma_2|} \tag{8}$$

This model still does not meet criterion 1, but it meets both criteria 2 and 3.

7. MODEL V: THE BERT FIBER-GOVERNED COMPLIANCE MODEL

This model was introduced by Bert [6] and differs from all of those previously discussed in that it relates stresses and strains in material-symmetry-axis coordinates (L, T) rather than principal-stress coordinates. Thus, instead of Eqs. (1), we have [6]:

$$\begin{aligned} \varepsilon_L &= b_{LL}\,\sigma_L + b_{LT}\,\sigma_T \\ \varepsilon_T &= b_{LT}\,\sigma_L + b_{TT}\,\sigma_T \\ \varepsilon_S &= b_{SS}\,\sigma_S \end{aligned} \tag{9}$$

where subscript S refers to shearing action relative to the L, T axes, and

$$b_{IJ} = \begin{cases} b_{IJ}^{(+)} & \text{if } \sigma_{fL} \geq 0 \\ b_{IJ}^{(-)} & \text{if } \sigma_{fL} < 0 \end{cases} \quad IJ = LL, LT, TT \tag{10}$$

and b_{SS} is independent of stress. Here σ_{fL} refers to the fiber stress in the fiber direction. A variety of approaches, based on different micromechanics assumptions, can be used to approximate the critical angle, i.e. the angle at which an applied uniaxial stress system must be oriented (with respect to the fiber direction) to achieve $\sigma_{fL} = 0$. Two such approaches were presented in [6]. However, it is not necessary to know this angle <u>à priori</u>; the test data can be reduced directly in such a way that the critical angle can be found directly [6].

In view of the success of this model in predicting the behavior of such a severely bimodular material as aramid-rubber (see Table 1), it is believed that this model comes closer to meeting criterion 1 than any of the other models proposed to date. Also, it meets criteria 2 and 3.

It should be noted that the Voigt or rule-of-mixtures model is an excellent micromechanics model for representing the stresses and modulus in the fiber direction for a single layer of filamentary composite material. This model is based upon the assumption that the fiber-direction strain in the fiber, matrix and composite all coincide. This affords a simple, practical criterion (the strain in the fiber direction) to determine the sign of the fiber stress at any given point in a laminated-composite-material structure.

8. FURTHER DISCUSSION

Three other unsymmetric compliance matrix approaches have been used in reduction of experimental data. As an alternative to Model V, Bert [6] also applied a nonsymmetric monomodulus model in reducing aramid-rubber data. However, he demonstrated that this model was not as satisfactory as Model V. Jones [19] applied both the original Ambartsumyan model (Model I) and an unsymmetric orthotropic bimodulus modification of the Jones-Nelson nonlinear model [20] in reducing biaxially-stressed ATJ-S graphite data. He also specifically studied the significance of lack of compliance-matrix symmetry and found that for the material considered only 1 to 2% difference could be attributed to this effect. Although in [5], Jones considered compliance-matrix symmetry a necessary criterion for a consistent material model, in [19] he considered the unsymmetric compliance matrix to be "correct". Regardless of the validity of an unsymmetric compliance matrix, since it makes only 1 to 2 % difference, it is clear that for practical engineering structural computations, a symmetric matrix is to be desired.

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PART II.

MICROMECHANICS OF THE DIFFERENT ELASTIC BEHAVIOR OF FILAMENTARY COMPOSITES IN TENSION AND COMPRESSION*

ABSTRACT

It has been known for a long time that certain kinds of fiber-reinforced composites exhibit quite different elastic and thermal-expansion characteristics depending upon whether the fibers are in tension or compression. The most dramatic differences have been noted for tire cord/rubber composites. Two different micromechanistic approaches have been proposed to explain these observed differences in tension/compression behavior: the mean-fiber-angle approach and the elastically supported tie-bar/column approach. In this paper, the previous work along both of these lines is reviewed and extended.

NOMENCLATURE

A	= fiber cross-sectional area
C	= constant of proportionality between ℓ_{cr} and d
d	= fiber diameter
E_f	= axial Young's modulus of the fibers
E_1	= major (fiber-direction) Young's modulus of the composite
F	= axial force in fiber
f, f_o	= βW, $\beta_o W_o$
G_m	= shear modulus of the matrix material
G_{12}	= shear modulus of the composite relative to 1, 2 axes
I	= area moment of inertia of fiber cross section
$I_1,\ldots,I_4$	= invariants defined in equations (37)
k	= foundation modulus of matrix material
k_1, k_2	= fiber-direction and transverse thermal conductivities
ℓ	= half-wave length of fiber deflection modal shape
M	= bending moment acting on a fiber
m, n	= $\cos\theta$, $\sin\theta$
S_{ij}	= elastic compliance coefficients ($i,j = 1,2,6$)
V	= transverse shear force acting on a fiber
W, W_o	= amplitudes of w and w_o, respectively
w, w_o	= fiber lateral deflection due to loading, initial fiber displacement from a straight line
x, y	= position coordinates parallel and normal to nominal fiber direction
α	= thermal-expansion coefficient
α_f, α_m	= α of fiber and matrix constituents, respectively

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* This part of the report is to be presented at the Symposium on Mechanics of Bimodulus Materials, sponsored by the Applied Mechanics Division, at the ASME Winter Annual Meeting, New York, NY, Dec. 1979, and will be published in the symposium volume.

β, β_0 = fiber sinusoidal parameter and its initial value
ΔT = temperature change from the strain-free temperature
ϵ_1, ϵ_2 = fiber-direction and transverse strains, total
$\epsilon_1^{(m)}$ = measured fiber-direction strain
σ = axial normal stress on fiber
ν_m = Poisson's ratio of the matrix material
ν_{12}, ν_{23} = major (axial-transverse) and transverse-transverse Poisson's ratios
θ = angle between local fiber direction and the nominal fiber direction

Superscripts:

$(\ )'$ d ( )/dx
$(\tilde{\ })$ indicates that this is a property of the wavy-fiber composite
$(\bar{\ })$ indicates that this is a mean value

Subscript:

cr denotes critical condition at which onset of buckling occurs

INTRODUCTION

In 1864, Saint-Venant (1)[1] presented a mechanics-of materials analysis of a beam made of a material having a different stress-strain relation when loaded in tension than when loaded in compression. Although this phenomenon has been observed in certain porous cellular media such as sintered metals (2), it has been observed most dramatically in fiber-reinforced materials, either man-made (3-6) or biological (7); see Table 1.

A number of mechanistic models have been proposed to explain the different tension-compression behavior for loading in the fiber direction. All of these models are based on the assumption that there is some initial curvature in the fibers, so that they are straightened and thus stiffened by tensile loading and curved more and thus made less stiff by compressive loading. All of the mechanistic models may be characterized into two general classes:

1. Tie-bar/column on elastic foundation model
2. Mean-fiber-angle approach

Table 1. Some Examples of Different Elastic Behavior in Tension And Compression

Investigator	Ref.	Material	E_c/E_t
Kotlarski & Karbasova	(3)	Fabric/rubber	0.38
Ducheyne et al.	(2)	Sintered, porous stainless steel	0.1
Zolotukhina & Lepetov	(4)	Various fabric/rubber	0.07 to 0.50
Patel el al.	(5)	Polyester cord/rubber	0.017
		Aramid cord/rubber	0.0034
Pósfalvi	(6)	Rayon cord/rubber	0.0036
Pearsall & Roberts	(7)	Myometrium (uterine muscle)	0.2

In the ensuing sections both of these models are described, critiqued, and extended.

[1] Underlined numbers in parentheses designate References at end of paper.

METHOD A: TIE-BAR/COLUMN ON ELASTIC FOUNDATION

Review of the Literature

Although earlier analyses had been concerned with the fiber buckling under compressive loading (8-10), perhaps the first analysis which specifically addressed the different fiber-direction stiffnesses in tension and compression was due to Herrmann et al. (11) in 1967. They were concerned with a relatively isolated wire in a flexible elastic medium, i.e. a low-fiber-volume-fraction (V_f) one. The application with which they were concerned was a wire-reinforced solid-propellant rocket grain. They developed two different analytical procedures, both of which considered the matrix as a three-dimensional elastic solid.

Other analyses of the different tension/compression (T/C) behavior of filamentary composites have been concerned with more closely spaced unidirectional composites. Both Bazant (12) and Swift (13) used two-dimensional models analogous to those used by Rosen (9) and Chung and Testa (14) for the study of fiber buckling. Of course there are two distinct fiber-buckling modes: extensional and shear.

It has been shown by Lager and June (15), Greszczuk (16), Davis (17), and Kulkarni et al. (18) among others, that advanced composites, consisting of boron, graphite, or aramid fibers in an epoxy matrix, under compressive loading fail in the shear buckling mode rather than the extensional one. However, these same composites are not the ones which exhibit large differences in T/C behavior at relatively small strains. These latter ones are characterized by low V_f. A simple analysis more appropriate to the latter case is presented in the next section.

Before leaving the topic of fiber buckling of high-V_f composites, it should be mentioned that Hayashi (19), Davis (17), and Wang (20) all found it necessary to take into account the effect of axial compressive stress on the shear modulus in order to obtain good agreement with experimental results. Also, the recent work of Schaffers (21) on intralaminar fiber buckling under *cylindrical bending* should be mentioned.

Recently Baer and associates (22-24) have conducted extensive research on uncalcified collagen tissues and have developed a mechanistic model based on the theory of an extensible elastica. However, their model does not take into account the support provided by the surrounding soft tissue (matrix material).

A Simple Model

The analysis presented here can be considered as an extension of the work of (11) to include thermal-expansion/contraction effects due to temperature change, with a considerably simplified expression for the foundation modulus (8).

The equations of equilibrium are:

$$F' = 0 \quad (1)$$

$$V = kw \quad (2)$$

$$M' - V + F(w_0 + w)' = 0 \quad (3)$$

Here $F \equiv$ axial force ($\pm$ for tension/compression), $k \equiv$ elastic foundation modulus for the matrix, $M \equiv$ bending moment, $V \equiv$ transverse shear force, $w \equiv$ lateral deflection of fiber, $w_0 \equiv$ initial deflection, $(\ )' \equiv d(\ )/dx$, and $x \equiv$ axial position coordinate.

Assuming the fiber to be a Bernoulli-Euler linear elastic member, the constitutive relations are:

$$M = -E_f I w'' \quad (4)$$

$$F = AE_f [\epsilon - (\alpha_f - \alpha_m) \Delta T] \quad (5)$$

Here $A \equiv$ fiber cross-sectional area, $E_f \equiv$ Young's modulus of the fiber along

its axis, $I \equiv$ area moment of inertia of fiber about its centroid, $\Delta T \equiv$ temperature change measured from a strain-free temperature, α_f and $\alpha_m \equiv$ fiber and matrix lineal coefficients of thermal expansion.

In view of equation (1), F is constant. Combining equations (2), (3), and (4), one obtains the following governing equation:

$$E_f I w^{IV} - F w'' + kw = F w_o'' \tag{6}$$

It is very likely that the initial deflection (w_o) is caused by a buckling of the fibers. Thus, it is reasonable to assume the same modal shape (dimensionless deflection distribution) for the initial deflection and deflection under loading.

$$w_o = W_o \sin \beta_o x \quad , \quad w = W \sin \beta_o x \tag{7}$$

Here W_o and W are the respective amplitudes of w_o and w, $\beta_o \equiv \pi/\ell$, and $\ell \equiv$ half-wave length of deflection modal shape.

Substituting w_o and w from equations (7) into equation (6), one obtains the following result:

$$W/W_o = - F \beta_o^2/(E_f I \beta_o^4 + F \beta_o^2 + k) \tag{8}$$

Finally, the following kinematic relationship is used to obtain an expression relating the fiber-direction strain ε, to the deflection amplitudes:

$$\varepsilon_1 = (\beta_o/\pi) \int_0^{\pi/\beta_o} \ell' \, dx \tag{9}$$

Here

$$\ell' = (1/2) [(w_o')^2 - (w')^2] + (F/AE_f) \tag{10}$$

Substituting equation (10) into equation (9) and integrating, one obtains

$$\varepsilon_1 = (\sigma/E_f) + (\alpha_m - \alpha_f) \Delta T + (\beta_o^2/4) (W_o^2 - W^2) \tag{11}$$

Finally, substituting W/W_o from equation (8) into equation (11), one obtains:

$$\varepsilon_1 = (\sigma/E_f) + (\alpha_m - \alpha_f)\Delta T + (\beta_o W_o/2)^2 \left\{ 1 - \left[\frac{AE_f[(\sigma/E_f) + (\alpha_m - \alpha_f)\Delta T]\beta_o^2}{E_f I \beta_o^4 + AE_f[(\sigma/E_f) + (\alpha_m - \alpha_f)\Delta T]\beta_o^2 + k} \right]^2 \right\} \tag{12}$$

<u>Special Cases</u>

1. No initial curvature ($\beta_o W_o \equiv 0$): Then equation (12) reduces to the following linear relation:

$$\varepsilon_1 = (\sigma/E_f) + (\alpha_m - \alpha_f) \Delta T \tag{12a}$$

This is the linear thermoelastic constitutive relation. Since the residual thermal strain $(\alpha_m - \alpha_f) \Delta T$ is not ususally measured, it is customary in tensile tests to reference the strain from the value of zero at the beginning of the test. Thus, the measured strain is

$$\varepsilon_1^{(m)} = \sigma/E_f \tag{12b}$$

2. High tension stress ($\sigma \sim$ large): Then ε_1 approaches the straight line of equation (12a) asymptotically from the right.
3. Zero stress ($\sigma \equiv 0$): Even when $\Delta T \equiv 0$ also, ε_1 is positive.
4. High compression stress ($- \varepsilon \sim$ large): The limiting case is

$$E_f I \beta^4 + F \beta^2 + k \to 0$$

thus,

$$-F = E_f I \beta^2 + k\,\beta^{-2} \tag{13}$$

Differentiating equation (13) with respect to β, one finds the initial value of β to be

$$\beta_{cr} = (k/E_f I)^{1/4} \tag{14}$$

Since $I = \pi d^4/64$, where d is the fiber diameter, equation (14) implies

$$\beta_{cr}\, d = \text{const.}$$

or

$$\ell_{cr} = C\, d \tag{15}$$

This result is consistent with the experiments of Rosen (9) and the elegant analysis of Sadowsky et al. (25).

To evaluate the constant C , we apply the following expression for the elastic foundation modulus of an infinite, isotropic medium surrounding a rigid fiber (8):

$$k = 16\pi G_m/[1 + 6(1 - 2\nu_m)] \tag{16}$$

For a rubber matrix, $\nu_m \approx 1/2$. Then

$$k \approx 16\pi G_m \tag{16a}$$

Thus

$$\beta_{cr}\, d = 2\sqrt{8}\,(G_m/E_f)^{1/4} \tag{17}$$

and

$$|-\sigma|_{cr} = 4(G_m E_f)^{1/2} \tag{18}$$

5. Decrease in temperature only ($\sigma \equiv 0$, $\Delta T < 0$): For $\Delta T = 0$, $\epsilon_1 > 0$. Since $\alpha_m > \alpha_f$ for practical composites, as ΔT is decreased ($\Delta T < 0$), ϵ_1 also decreases. The limiting case is when

$$E_f I \beta^4 + AE_f\,(\alpha_m - \alpha_f)\beta^2\,\Delta T + k \to 0$$

Then

$$|-\Delta T| = (E_f I \beta^2 + k\,\beta^{-2})/AE_f\,(\alpha_m - \alpha_f) \tag{19}$$

This has a critical value of β given by equation (17) and

$$|-\Delta T|_{cr} = (k/E_f)^{1/2}/(\alpha_m - \alpha_f) \approx 4(\pi G_m/E_f)^{1/2}/(\alpha_m - \alpha_f) \tag{20}$$

METHOD B: MEAN-FIBER-ANGLE APPROACH

Review of the Literature

Although an earlier analysis was concerned with initially wavy layered media (26), the first analysis specifically addressed to the use of this approach to fiber-reinforced composites was due to Tarnopol'skii et al. (27). They assumed that the fibers are initially sinusoidally curved, equation (7a). Then the inclination at an arbitrary axial position x is

$$\theta = \arctan\,(\beta_o W_o \cos \beta_o x) \tag{21}$$

Using equation (21) in the well-known transformation equations for Young's modulus and integrating over a half-wave length ℓ, they obtained the following result for the fiber-direction initial Young's modulus of a wavy-fiber-reinforced composite:

$$(1/\tilde{E}_1) = (1/2E_1)(2 + f_o^2)(1 + f_o^2)^{-3/2}$$
$$+ (G_{12}^{-1} - 2\nu_{12}E_1^{-1})(f_o^2/2)(1 + f_o^2)^{-3/2}$$
$$+ (1/2E_2)[2 - (2 + 3f_o^2)(1 + f_o^2)^{-3/2}] \tag{22}$$

Here E_1 and E_2 are the major (fiber direction) and minor Young's moduli of a unidirectional composite with straight fibers, $\tilde{E}_1$ is the major Young's modulus of the wavy-fiber composite, $f_o \equiv \beta_o W_o$, and G_{12} and ν_{12} are the respective major shear modulus and major Poisson's ratio of the straight-fiber composite.

For $f_o << 1$, the following approximate expression was suggested in (27):

$$\tilde{E}_1/E_1 \approx [1 + (f_o^2/2)(E_1/G_{12})]^{-1} \tag{23}$$

This expression is consistent with Bolotin's result (26).

The disadvantage of using either equation (14) or (15) is that it requires prior knowledge of a straight-fiber composite having the same fiber volume fraction. For the glass-fiber reinforced plastic investigated experimentally in (27), it was found empirically that an initial composite prestress of approximately 10% of the ultimate tensile strength was necessary to assure that the fibers are sufficiently straight.

In an alternate approach to that of (27), Nosarev (28) considered the curved fiber as a finite number of straight-line segements. He obtained numerical results which predicted reasonably well the decrease in major Young's modulus and increase in major Poisson's ratio for a copper-wire-reinforced epoxy composite as the fiber curvature is increased.

The work of (26-28) can be used to predict only the initial modulus (or Poisson's ratio). However, in certain composites with a highly flexible matrix (such as rubber), the composite modulus may change quite significantly with loading (especially in compression, as discussed in the preceding section). Thus, the analysis of Tabaddor and Chen (29) is quite important for such composites, since it permits computation of the complete stress-strain curve in both tension and compression for the composite in the fiber direction. In their work, they replaced f_o in equation (22) with f ($\equiv \beta W$) and developed a relationship between f and f_o.

Makarov and Nikolaev (30) conducted a series of experiments on aluminum-wire-reinforced natural rubber in which the wires were provided intentionally with controlled amounts of initial curvature. The ratio of Young's moduli of fiber and matrix was 13,700 and the fiber volume fraction was 0.06. They found that curvature had a negligible effect on the minor Young's modulus ($\tilde{E}_2$), the transverse-transverse Poisson's ratio ($\tilde{\nu}_{23}$), and the transverse thermal-expansion coefficient ($\tilde{\alpha}_2$). However, for their test conditions they measured approximately 50% decrease in $\tilde{E}_1$, 48% increase in $\tilde{\nu}_{12}$, and 15% increases in $\tilde{G}_{12}$ and $\tilde{\alpha}_1$.

Vishnevskii and Shlenskii (31) investigated the effect of sinusoidally curved fibers on transverse thermal conductivity ($\tilde{k}_2$) of the composite. They derived an expression for the transverse thermal conductivity which is equivalent to the following one in the present notation:

$$\tilde{k}_2 = (1/2)[k_1 f_o^2 + k_2(2 - f_o^2)] \tag{24}$$

Here k_1 and k_2 are the conductivities in the fiber and transverse directions.

It should be mentioned here that the companion equation to equation (24) can be written down immediately, from symmetry, as follows:

$$\tilde{k}_1 = (1/2)[k_1(2 - f_o^2) + k_2 f_o^2] \tag{25}$$

Also, it is noted here that, in view of the fact that the angular transformation equations for thermal expansion are identical (second-rank tensor) to those for thermal conductivity, equations (24) and (25) are also applicable to thermal expansion provided that the k's are replaced by α's.

Tarnopol'skii et al. (32) investigated the effect of fiber waviness in composites with anisotropic fibers (specifically carbon fibers). They found that for such composites, it is necessary to use the full equation (22) rather than the simplified equation (23). Van Dreumel and Kamp (33) investigated experimentally the effects of fiber waviness on both tensile strength and major Poisson's ratio of carbon-fiber reinforced plastic.

In order to completely characterize a thin sheet material for use in stress analysis and structural design, one needs a complete set of elastic stress-strain relations, for fiber-direction tension and compression (34-35), not just the Young's moduli. A rational analytical basis for such a complete characterization is developed for the first time in the following section.

Complete Characterization of Planar Compliances

A typical repeating element is assumed to consist of a segment of a sinusoidally curved fiber and its surrounding matrix material. The fiber curvature is assumed to be planar in a single plane (xy) with a path given by

$$y = W_0 \sin \beta_0 x \tag{26}$$

Following the general approach used by Tabaddor and Chen (29), it is assumed that when the fiber is subjected to a positive (tensile) strain in directions 1 and 2, respectively parallel and normal to the fiber direction, it takes on a new sinusoidal form given by

$$y = W \sin \beta x \tag{27}$$

where the relationship between β and β_0 and between W and W_0 are given by

$$(2\pi/\beta_0)(1 + \epsilon_1) = 2\pi/\beta \tag{28}$$

$$W_0(1 + \epsilon_2) = W \tag{29}$$

It is assumed that shear strain has no effect on these inter-relationships. Combining equations (28) and (29), one obtains

$$f = \beta W = \beta_0 W_0 \,(1 + \epsilon_2)/(1 + \epsilon_1) \tag{30}$$

In view of the definitions of f_0 and of Poisson's ratio ($\nu_{12} \equiv -\epsilon_2/\epsilon_1$), equation (30) can be rewritten as follows for the uniaxial case only

$$f = f_0 \,(1 - \nu_{12}\,\epsilon_1)/(1 + \epsilon_1) \tag{31}$$

Equation (31) expresses the effect of ϵ_1 on $f(= \beta W)$. This is a different approach than in the tie-bar/column one (Method A), in which it is assumed that $\beta = \beta_0$ and W_0 is found by solution of the governing differential equation.

The work that follows is based on the elastic compliance rather than the planar stiffnesses or the engineering moduli (Young's modulus, Poisson's ratio). There are several reasons for this choice:

1. Permits direct comparison with experimentally measured compliances.
2. Is not restricted to a particular stress state (such as uniaxial, biaxial, or even triaxial) but can be applied directly to any stress state.

The transformation equations relating the elastic compliance coefficients with respect to axes ($\bar{1}$, $\bar{2}$), oriented at an angle θ with respect to the orthotropic material-symmetry directions (1,2), to those with respect to (1, 2) are (36):

$$\begin{Bmatrix} \tilde{S}_{11} \\ \tilde{S}_{12} \\ \tilde{S}_{22} \\ \tilde{S}_{66} \end{Bmatrix} = \begin{bmatrix} S_{11} & 2S_{12} + S_{66} & S_{22} \\ S_{12} & S_{11} + S_{22} - S_{66} & S_{12} \\ S_{22} & 2S_{22} + S_{66} & S_{11} \\ S_{66} & 4S_{11} + 4S_{22} - 8S_{12} - 2S_{66} & S_{66} \end{bmatrix} \begin{Bmatrix} m^4 \\ m^2 - m^4 \\ 1 - 2m^2 + m^4 \end{Bmatrix}$$

$$\begin{Bmatrix} \tilde{S}_{16} \\ \tilde{S}_{26} \end{Bmatrix} = \begin{bmatrix} 2S_{11} - 2S_{12} - S_{66} & -2S_{22} + 2S_{12} + S_{66} \\ -2S_{22} + 2S_{12} + S_{66} & 2S_{11} - 2S_{12} - S_{66} \end{bmatrix} \begin{Bmatrix} m^3 n \\ mn^3 \end{Bmatrix} \tag{32}$$

Here $m \equiv \cos\theta$, $n \equiv \sin\theta$.

In writing equations (24), the contracted index notation was used. Subscripts 1 and 2 refer to normal stress/strain action in directions 1 and 2 respectively, while subscript 6 refers to shear stress/strain action related to axes 1 and 2.

To average the properties over one wavelength of fiber waviness, it is necessary to relate direction θ to the curved-fiber geometric properties W and β. Working toward this objective, we take the derivative of both sides of equation (27):

$$\tan\theta = y' = W\beta \cos \beta x \tag{33}$$

Thus,

$$\begin{aligned} m &= \cos\theta = [1 + (W\beta \cos \beta x)^2]^{-1/2} \\ n &= \sin\theta = W\beta \cos \beta x / [1 + (W\beta \cos \beta x)^2]^{1/2} \end{aligned} \tag{34}$$

The average values of the functions of m and n appearing in equations (32) are

$$\begin{aligned} \overline{m^4} &\equiv 2 \int_{-L/4}^{L/4} m^4 \, dx \Big/ 2 \int_{-L/4}^{L/4} dx = [1 + (f^2/2)](1 + f^2)^{-3/2} \\ \overline{m^2} &\equiv (1 + f^2)^{-1/2} \\ \overline{m^3 n} &= \overline{mn^3} = 0 \end{aligned} \tag{35}$$

It is now possible to express the curved-fiber composite compliances $\tilde{S}_{ij}$ in terms of the straight-fiber unidirectional compliances S_{ij} as follows:

$$\begin{Bmatrix} \tilde{S}_{11} \\ \tilde{S}_{12} \\ \tilde{S}_{22} \\ \tilde{S}_{66} \end{Bmatrix} = \begin{bmatrix} S_{22} & I_1 & I_2 \\ S_{12} & I_2 & -I_1 \\ S_{11} & I_3 & I_2 \\ S_{66} & I_4 & -I_4 \end{bmatrix} \begin{Bmatrix} 1 \\ \overline{m^2} \\ \overline{m^4} \end{Bmatrix} \tag{36}$$

Here the invariants are

$$\begin{aligned} I_1 &\equiv 2S_{12} - 2S_{22} + S_{66}, \quad I_2 \equiv S_{11} - 2S_{12} + S_{22} - S_{66} \\ I_3 &\equiv -2S_{11} + 2S_{12} + S_{66}, \quad I_4 \equiv 4S_{11} - 8S_{12} + 4S_{22} - 4S_{66} \end{aligned}$$

It is noted that since $\overline{m^3 n} = \overline{mn^3} = 0$, the wavy-fiber composite remains orthotropic. Use of equations (30), (35), and (36) in an iterative manner permits development of complete nonlinear strain versus stress relationships for the composite.

DISCUSSION AND CONCLUDING REMARKS

In both of the mechanistic analytical methods presented here, no provision was made for transverse shear deformation of the fibers. In fact, there is some controversy as to which is the best way to incorporate such action; see the work of Roze and Kintsis (37). However, a suitable one of the approaches described in (37) could be incorporated into Method A, if desired. It is noted that in Method B flexural, muchover shear, properties do not enter explicitly into the equations.

The experiments of Patel et al. (5) showed that the difference between tension and compression properties were much more pronounced for rubber reinforced with aramid cord than when reinforced with polyester cord. This is probably related to the very low compressive strength of the aramid fibers themselves, which Greenwood and Rose (38) suggested is due to separation of the microfibrils comprising the fibers.

Moncunill de Ferran and Harris (39) found experimentally that the buckling mode of steel wire/polyester was helical rather than planar as assumed in all known analyses. It has been suggested that the more pronounced difference between tensile and compressive behavior exhibited in the tire-cord/rubber composites of (5-6) as compared to that of ordinary wire-reinforced rubber (8) may be due to the inherent twisting action present in tire cord, which has a helical geometry (40). However, in (5), steel-cord/rubber was also investigated and no appreciable difference between the tensile and compressive stress-strain behavior was noted.

From the review and methods presented here, it can be concluded that the field of micromechanics of fiber-reinforced composites with different properties in tension and compression is in its infancy. Considerably more work needs to be done, among which the following appear to be the most pressing:

1. Definitive experiments to determine which of the two methods (A and B) presented here are the most successful in predicting composite material behavior (not only in uniaxial loading but also under multiaxial conditions) and to help refine or even combine them.
2. Experiments to determine the role of fiber-matrix interfacial adhesion on the gross behavior.
3. Fracture mechanics analyses and carefully controlled experiments with controlled flaws.
4. Analyses incorporating more realistic models of the helical-strand action in tire cords (40).
5. Simple means of incorporating the different T/C behavior into static and dynamic analyses of structural components such as plates and shells (41).

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